

Pig and Odd Pig Out

Topics: Probability, strategy, addition, multiplication, estimation

Materials: One 6-sided die per 2-4 players, pencil and paper

Time: 5-10 minutes to learn; 5 - 10 minutes to play; 1 - 3 lessons to explore

Roll the dice and collect points. You can go as long as you want, but roll the wrong number and you lose all your points from that turn!

Why We Love Pig

Pig is easy to learn and gives students practice with addition (and multiplication, with Odd Pig Out). Pig is mathematically rich. Students must articulate and defend strategies relating to handling chance.

The Launch

Before you start, remind students of the importance of winning and losing gracefully. They'll do a lot of both with Pig, and it's important to take it lightly, since it's so easy to have setbacks.

Pig is a game that will sometimes punish a good decision and reward a bad one, which presents a real challenge for students: how can you tell if you made a good decision or a bad decision? Does strategy even matter in a game of luck like Pig? The teacher can bring these questions out over the course of the lesson, and let students grapple with them. In the long term, taking a scientific approach by running experiments and collecting data is one excellent way to handle the problem.

Invite a volunteer to play a demonstration game. Make sure you take lots of risks, and let the students give you “thumbs up/down” if they think you should keep rolling.

How to Play

Pig is a game for 2 to 6 players. Players take turns rolling a die as many times as they like. If a roll is a 2, 3, 4, 5, or 6, the player adds that many points to their score for the turn. A player may choose to end their turn at any time and “bank” their points. If a player rolls a 1, they lose all their unbanked points and their turn is over.

Beginner Game: The first player to score 50 or more points wins.

Advanced Game: The first player to score 100 or more points wins.

Demonstrate enough turns so that students can see how rolling a 1 will lose them unbanked points, and that points in their bank will be safe even when a 1 is rolled.

The Work

Students can play Pig for fun anytime. Games can be quick and light.

The deeper work of Pig comes when we start to examine strategy. As students play, ask them to notice reflect on what strategy they're using as they as they play. Are they taking big risks, or is their play more conservative? After students have had enough time to play, discuss strategy for Pig as a class. What strategies did students use? Does strategy matter? How do you know? Pig is clearly a game of chance, but does that mean strategy makes no difference?

The stage is being set to actually run an experiment. How can we determine for sure whether strategy matters or not? We could pit two strategies against each other, and see which one wins; the more extreme the strategies, the more clearly we could see the difference. And what are the most extreme strategies? The most conservative we call *Better Safe than Sorry*: roll once and immediately bank your points. The most extreme we call *Let It Ride*: keep rolling until you get 50 points and win, or roll a 1 and lose all your points.

So if one person uses the *Better Safe than Sorry* strategy, and the other plays the *Let It Ride* strategy, who is more likely to win? Let students vote, and collect the number of votes from students as to which strategy they think will win. It might look like this:

<i>Better Safe than Sorry</i>	<i>Let It Ride</i>	No Difference
13	5	4

Now you can run an actual experiment. Have students play in pairs, each playing one of the opposing strategies. They should play to 50 points, and keep track of how many times each strategy wins. If students have 10-15 minutes to collect data, you'll likely have a good number of finished games. Collect all the data together on the board, and add up how many games were won for each strategy. In my experience, and quite surprisingly, *Let It Ride* tends to win about 75% of the time.

There's a big discussion here about probability, statistics, certainty, and uncertainty. Is the classroom data convincing to students? Has it settled the questions about whether strategy matters and which strategy is better? What would be a better strategy to pit against *Let It Ride* in a future game (for example, roll 3 times and then bank your points)? It's possible to run successive experiments, or to have students program a computer to run experiments for them.

And of course, you can always let students just play the game for fun when you have some extra time in class.

Prompts and Questions

- How long are you waiting before you stop rolling?
- Do you have a strategy?

- Before you roll again, tell me how many points you already have for this turn.
- What's the best way to add those numbers up?

The Wrap

In addition to the experimental approach described above, we can wrap up playing the game by discussing the probabilities of outcomes, and how they can help us make predictions for good moves. Consider these questions:

- What is the probability that you roll a 1 on a given roll? (Answer: $1/6$)
- What is the probability you won't roll a 1? (Answer: $5/6$)
- If you don't roll a 1, what is your average point gain? (Answer: 4)

Considering these values, you can reframe the question for each roll in the following way: is it worth risk losing the points you haven't banked ($1/6$ chance) yet in order to have a of gaining about 4 points ($5/6$ chance). That means your chances of gaining an average of 4 points on a given roll is 5 times greater than your chance of losing all your points. When is this worth it?

If you have 10 points unbanked, should you risk them at 5:1 odds in order to gain 4 more? That actually seems like a good bet. In fact, anything up to 20 points ($20 = 5 \times 4$) seems like a good bet when you're considering the problem in this framework. This gives a mathematical rule of thumb for how you might want to proceed. Another experiment to try, then, is to play to 100, but have some people try the "Bank when you have 20 points or more" strategy, and others play some other strategy of their choice. Who will tend to win? (The difference may be subtle.) Still, this is another example of how mathematical analysis can give us some control over a situation, even when there's a great element of chance involved.

Variation - Odd Pig Out

Materials: 2 dice per 2-4 players. Can use 6-, 8-, 10-, or 12- sided dice. The discussion below is for 6-sided dice.

Roll two dice instead of 1. If the product of the two dice is even, score the product and decide whether to roll again or bank your points for the turn. If the product is odd, lose your unbanked points and end your turn. Play to 300 points or more.

The same mathematical approach can give a good rule of thumb for a strategy for Odd Pig Out. Students will need to do the harder mathematical work of figuring out:

- What is the probability that you roll an odd product on a given roll? (Answer: $1/4$)
- What is the probability that you roll an even product? (Answer: $3/4$)
- If you don't roll an odd product what is your average point gain?
(Answer: the sum of the even numbers on a multiplication table of the appropriate size, divided by the number of even products. This is, in fact, a fascinating mathematical counting problem. For a 6 by 6 multiplication table, the sum of the even

numbers is $21^2 - 9^2 = 441 - 81 = 360$.

There are $36 - 9 = 27$ even products, so the average is $360/27 = 13\frac{1}{3}$.)

If we use the same argument as before, you should be willing to risk up to 40 points if you have 3 to 1 odds of winning $13\frac{1}{3}$ so that's a good estimate of how risky you should be willing to be in Odd Pig Out.

Tips for the classroom

1. Demonstrate the game a couple times with the whole group. Solicit advice about when you (the teacher) should stop rolling on your turn. Students can give you a thumbs up if they think you should continue rolling, and a thumbs down if they think you should stop.
2. Remind students that they will lose games and win games, and each loss can be a chance to re-examine how they are playing. It's hard to lose all your points, but it will happen to everyone!
3. As students play each other, circulate through the room and ask them about their strategies. It's ok for students simply to play, but there's an opportunity to probe deeper into the workings of chance and the strategy of the game too.

Name _____

Pig

Rolls

Rolls

Bank

Bank