6.6. Core Processes: Reasoning, problem solving, and communication

Students refine their reasoning and problem-solving skills as they move more fully into the symbolic world of algebra and higher-level mathematics. They move easily among representations—numbers, words, pictures, or symbols—to understand and communicate mathematical ideas, to make generalizations, to draw logical conclusions, and to verify the reasonableness of solutions to problems. In grade six, students solve problems that involve fractions and decimals as well as rates and ratios in preparation for studying proportional relationships and algebraic reasoning in grade seven.

Mathematics content based on
Adopted Washington State
K-8 Mathematics Standards,
April 28, 2008,
Office of the Superintendent of Public Instruction

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Grade 6 Mathematics Standards

Adopted Washington State
April 28, 2008

6.1. Core Content: Multiplication and division of fractions and decimals (Numbers, Operations, Algebra)

Students have done extensive work with fractions and decimals in previous grades and are now prepared to learn how to multiply and divide fractions and decimals with understanding. They can solve a wide variety of problems that involve the numbers they see every day—whole numbers, fractions, and decimals. By using approximations of fractions and decimals, students estimate computations and verify that their answers make sense.

6.2. Core Content: Mathematical expressions and equations (Operations, Geometry/ Measurement, Algebra)

Students continue to develop their understanding of how letters are used to represent numbers in mathematics—an important foundation for algebraic thinking. Students use tables, words, numbers, graphs, and equations to describe simple linear relationships. They write and evaluate expressions and write and solve equations. By developing these algebraic skills at the middle school level, students will be able to make a smooth transition to high school mathematics.

6.3. Core Content: Ratios, rates, and percents (Numbers, Operations, Geometry/Measurement, Data/Statistics/Probability)

Students extend their knowledge of fractions to develop an understanding of what a ratio is and how it relates to a rate and a percent. Fractions, ratios, rates, and percents appear daily in the media and in everyday calculations like determining the sale price at a retail store or figuring out gas mileage. Students solve a variety of problems related to such situations. A solid understanding of ratios and rates is important for work involving proportional relationships in grade seven.

6.4. Core Content: Two-dimensional figures (Geometry/ Measurement, Algebra)

Students extend what they know about area and perimeter to more complex two-dimensional figures, including circles. They find the surface area and volume of simple three-dimensional figures. As they learn about these important concepts, students can solve problems involving more complex figures than in earlier grades and use geometry to deal with a wider range of situations. These fundamental skills of geometry and measurement are increasingly called for in the workplace and they lead to a more formal study of geometry in high school.

6.5. Additional Key Content (Numbers, Operations)

Students extend their mental math skills now that they have learned all of the operations—addition, subtraction, multiplication, and division—with whole numbers, fractions, and decimals. Students continue to expand their understanding of our number system as they are introduced to negative numbers for describing positions or quantities below zero. These numbers are a critical foundation for algebra, and students will learn how to add, subtract, multiply, and divide positive and negative numbers in seventh grade as further preparation for algebraic study.

Grade Six Performance Expectations . . . . . .

Multiplication and Division of Fractions and Decimals (numbers, operations, algebra)

6.1.A **Compare and order non-negative fractions, decimals,** *and integers* **using the number line, lists, and the symbols <, >, or =.** (4.2.E)

6.1.B *Represent multiplication and division of non-negative fractions and decimals using area models and the number line, and connect each representation to the related equation.* (7.1.B)

6.1.C **Estimate products and quotients of fractions and decimals.** ** (5.1.D)

6.1.D *Fluently and* **accurately multiply and divide non-negative fractions** *and explain the inverse relationship between multiplication and division with fractions.* (7.1.C)

6.1.E **Multiply and divide whole numbers and decimals by 1000, 100, 10, 1, 0.1, 0.01, and 0.001.** (5.1.B)

6.1.F *Fluently and* **accurately multiply and divide non-negative decimals.** (4.1.F) (5.1.C) (7.1.C)

6.1.G **Describe the effect of multiplying or dividing a number by one, by zero, by a number between zero and one, and by a number greater than one.** **

6.1.H **Solve single- and multi-step word problems involving operations with fractions and decimals and verify the solutions.** (5.2.H) (7.1.G)

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** This performance expectation may be included in items assessing process performance expectaions.

Mathematical Expressions and Equations (operations, geometry/measurement, algebra)

6.2.A **Write a mathematical expression or equation with variables to represent information in a table or given situation.** (5.4.B) (5.4.C) (7.1.F)

6.2.B **Draw a first-quadrant graph in the coordinate plane to represent information in a table or given situation.** (5.4.D)

6.2.C **Evaluate mathematical expressions when the value for each variable is given.** (5.4.C)

6.2.D **Apply the commutative, associative, and distributive properties, and use the order of operations to evaluate mathematical expressions.** (8.4.C)

6.2.E **Solve one-step equations and verify solutions.** (7.1.E)

6.2.F **Solve word problems using mathematical expressions and equations and verify solutions.**

Ratios, rates, and percents (numbers, operations, geometry/measurement, data/statistics/probability)

6.3.A **Identify** *and write* **ratios as comparisons of part-to-part and part-to-whole relationships.**

6.3.B **Write ratios to represent a variety of rates**.

6.3.C **Represent percents visually and numerically, and convert between the fractional, decimal, and percent representations of a number.**

6.3.D **Solve single- and multi-step word problems involving ratios, rates, and percents, and verify the solutions.**

6.3.E *Identify the ratio of the circumference to the diameter of a circle as the constant π, and recognize 22/7 and 3.14 as common approximations of pi.*

6.3.F **Determine the experimental probability of a simple event using data collected in an experiment.** (4.4.H)

6.3.G **Determine the theoretical probability of an event and its complement and represent the probability as a fraction or decimal from 0 to 1 or as a percent from 0 to 100.** (4.4.G) (7.4.A) (7.4.B)

Two-Dimensional Figures (geometry/measurement, algebra)

6.4.A **Determine the circumference and area of circles.** (7.3.A)

6.4.B **Determine the perimeter and area of a composite figure that can be divided into triangles, rectangles, and parts of circles.** (5.3.F)

6.4.C **Solve single- and multi-step word problems involving the relationships among radius, diameter, circumference, and area of circles, and verify the solutions**. (7.2.B) (7.3.D)

6.4.D **Recognize** *and draw* **two-dimensional representations of three-dimensional figures.**

6.4.E **Determine the surface area and volume of rectangular prisms using appropriate formulas** *and explain why the formulas work.* (7.3.A)

6.4.F **Determine the surface area of a pyramid.** (7.3.B)

6.4.G **Describe and sort polyhedra by their attributes: parallel faces, types of faces, number of faces, edges, and vertices.**

Additional Key Content (numbers, operations)

6.5.A *Use strategies for mental computations with non-negative whole numbers, fractions, and decimals.* (5.1.E) (7.2.A)

6.5.B **Locate positive and negative integers on the number line and use integers to represent quantities in various contexts.**

6.5.C **Compare and order positive and negative integers using the number line, lists, and the symbols <, >, or =.** (4.2.E) (7.1.A)

Reasoning, Problem Solving, and Communication

6.6.A *Analyze a problem situation to determine the question(s) to be answered.* (5.6.A) (7.6.A)

6.6.B **Identify relevant, missing, and extraneous information related to the solution to a problem.** (5.6.B) (5.6.C) (7.6.B)

6.6.C *Analyze and compare mathematical strategies for solving problems, and* **select and use one or more strategies to solve a problem.** (5.6.D) (5.6 E) (7.6.C)

6.6.D **Represent a problem situation,** *describe the process used to solve the problem, and* **verify the reasonableness of the solution**. (5.6.F) (5.6.G) (5.6.H) (7.6.D)

6.6.E *Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.* (5.6.I) (7.6.E)

6.6.F *Apply a previously used problem-solving strategy in a new context.* (5.6.D) (7.6.F)

6.6.G **Extract and organize mathematical information from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.** (5.6.I) (7.6.G)

6.6.H *Make and test conjectures based on data (or information) collected from explorations and experiments.* (5.6.J)(7.6.H)

7.6. Core Processes: Reasoning, problem solving, and communication

Students refine their reasoning and problem-solving skills as they move more fully into the symbolic world of algebra and higher-level mathematics. They move easily among representations—numbers, words, pictures, or symbols—to understand and communicate mathematical ideas, to make generalizations, to draw logical conclusions, and to verify the reasonableness of solutions to problems. In grade seven, students solve problems that involve positive and negative numbers and often involve proportional relationships. As students solve these types of problems, they build a strong foundation for the study of linear functions that will come in grade eight.

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Grade 7 Mathematics Standards . . . . . .

Adopted Washington State

April 28, 2008

7.1. Core Content: Rational numbers and linear equations (Numbers, Operations, Algebra)

Students add, subtract, multiply, and divide rational numbers—fractions, decimals, and integers—including both positive and negative numbers. With the inclusion of negative numbers, students can move more deeply into algebraic content that involves the full set of rational numbers. They also develop the algebraic skill of solving equations that require more than one step, allowing them to approach problems that deal with a wider range of contexts than before. Using generalized algebraic skills and approaches, students can pursue a wide range of problems involving any type of rational number, adapting strategies for solving one problem to different problems in different settings with underlying similarities.

7.2. Core Content: Proportionality and similarity (Operations, Geometry/Measurement, Algebra)

Students extend their work with ratios to solve problems involving a variety of proportional relationships, such as making conversions between measurement units or finding the percent increase or decrease. They also solve problems involving the proportional relationships related to similar figures, and in so doing reinforce an important connection between numerical operations and geometric relationships. Students graph proportional relationships and identify the rate of change as the slope of the related line. The skills and concepts related to proportionality represent some of the most important connecting ideas across K–12 mathematics. With a good understanding of how things grow proportionally, students can understand the linear relationships that are the basis for high school mathematics. If learned well, proportionality can open the door for success in much of secondary mathematics.

7.3. Core Content: Surface area and volume (Geometry/Measurement)

Students extend their understanding of area and perimeter to finding the surface area and volume of three-dimensional figures. They apply formulas and solve a range of problems involving three-dimensional objects, including problems people encounter in everyday life, in certain types of work, and in other school subjects. With a strong understanding of how to work with both two-dimensional and three-dimensional figures, students have an important base for the geometry they will study in high school.

7.4. Core Content: Probability and data (Data/Statistics/Probability)

Students apply their understanding of rational numbers and proportionality to concepts of probability. They begin to understand how probability is determined, and they make related predictions. Students revisit how to interpret data, now using sophisticated types of data graphs and thinking about the meaning of certain statistical measures. Statistics, including probability, is considered one of the most important and practical fields of study for making sense of quantitative information, and it plays an important part in secondary mathematics in the 21st century.

7.5. Additional Key Content (Numbers, Algebra)

Students extend their coordinate graphing skills to plotting points with both positive and negative coordinates on the coordinate plane. Using pairs of numbers to locate points is a necessary skill for reading maps and tables and a critical foundation for high school mathematics. Students further prepare for algebra by learning how to use exponents to write numbers in terms of their most basic (prime) factors.

Grade Seven Performance Expectations

Rational Numbers and Linear Equations (numbers, operations, algebra)

7.1.A **Compare and order rational numbers using the number line, lists, and the symbols <, >, or =.** (6.5.C)

7.1.B *Represent addition, subtraction, multiplication, and division of positive and negative integers visually and numerically.* (6.1.B)

7.1.C *Fluently and* **accurately add, subtract, multiply, and divide rational numbers.** (6.1.D) (6.1.F)

7.1.D *Define and* **determine the absolute value of a number.**

7.1.E **Solve two-step linear equations.** (6.2.E) (8.1.A)

7.1.F **Write an equation that corresponds to a given problem situation,** *and describe a problem situation that corresponds to a given equation.* (6.2.A)

7.1.G **Solve single- and multi-step word problems involving rational numbers and verify the solutions.** (6.1.H)

Proportionality and Similarity (operations, geometry/measurement, algebra)

7.2.A *Mentally add, subtract, multiply, and divide simple fractions, decimals, and percents.* (6.5.A)

7.2.B **Solve single- and multi-step problems involving proportional relationships and verify the solutions.** (6.4.C)

7.2.C **Describe proportional relationships in similar figures and solve problems involving similar figures.**

7.2.D *Make scale drawings and* **solve problems related to scale.**

7.2.E **Represent proportional relationships using graphs, tables, and equations, and make connections among the representations.**

7.2.F **Determine the slope of a line corresponding to the graph of a proportional relationship** *and relate slope to similar triangles.* (5.4.D)

7.2.G **Determine the unit rate in a proportional relationship and relate it to the slope of the associated line.** **

7.2.H *Determine whether or not a relationship is proportional and explain your reasoning.*

7.2.I **Solve single- and multi-step problems involving conversions within or between measurement systems and verify the solutions.**

Surface Area and Volume (geometry/measurement)

7.3.A **Determine the surface area and volume of cylinders using the appropriate formulas** *and explain why the formulas work.* (6.4.A) (6.4.E)

7.3.B **Determine the volume of pyramids and cones using formulas.** (6.4.F)

7.3.C **Describe the effect that a change in scale factor on one attribute of a two- or three-dimensional figure has on other attributes of the figure, such as the side or edge length, perimeter, area, surface area, or volume of a geometric figure.**

7.3.D **Solve single- and multi-step word problems involving surface area or volume and verify the solutions.** (6.4.C)

Probability and Data (data/statistics/probability)

7.4.A **Represent the sample space of probability experiments in multiple ways, including tree diagrams and organized lists.** (6.3.G)

7.4.B **Determine the theoretical probability of a particular event and use theoretical probability to predict experimental outcomes.** (6.3.G) (8.3.F)

7.4.C **Describe a data set using measures of center (median, mean, and mode) and variability (maximum, minimum, and range)** *and evaluate the suitability and limitations of using each measure for different situations.* (5.5.B) (8.3.A)

7.4.D **Construct and interpret histograms, stem-and-leaf plots, and circle graphs.** (8.3.B)

7.4.E **Evaluate different displays of the same data for effectiveness and bias, and explain reasoning.** ** (8.3.D) (8.3.E)

Additional Key Content (numbers, algebra)

7.5.A **Graph ordered pairs of rational numbers and determine the coordinates of a given point in the coordinate plane.** (5.4.D)

7.5.B **Write the prime factorization of whole numbers greater than 1, using exponents when appropriate.** (5.2.D) (5.5.A)

Reasoning, Problem Solving, and Communication

7.6.A *Analyze a problem situation to determine the question(s) to be answered.* (6.6.A) (8.5.A)

7.6.B **Identify relevant, missing, and extraneous information related to the solution to a problem.** (6.6.B) (8.5.B)

7.6.C *Analyze and compare mathematical strategies for solving problems, and* **select and use one or more strategies to solve a problem.** (6.6.C) (8.5.C)

7.6.D **Represent a problem situation,** *describe the process used to solve the problem, and* **verify the reasonableness of the solution.** (6.6.D) (8.5.D)

7.6.E *Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.* (6.6.E) (8.5.E)

7.6.F *Apply a previously used problem-solving strategy in a new context.* (6.6.F) (8.5.F)

7.6.G **Extract and organize mathematical information from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.** (6.6.G) (8.5.G)

7.6.H *Make and test conjectures based on data (or information) collected from explorations and experiments.* (6.6.H) (8.5.H)

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8.5. Core Processes: Reasoning, problem solving, and communication

Students refine their reasoning and problem-solving skills as they move more fully into the symbolic world of algebra and higher-level mathematics. They move easily among representations—numbers, words, pictures, or symbols—to understand and communicate mathematical ideas, to make generalizations, to draw logical conclusions, and to verify the reasonableness of solutions to problems. In grade eight, students solve problems that involve proportional relationships and linear relationships, including applications found in many contexts outside of school. These problems dealing with proportionality continue to be important in many applied contexts, and they lead directly to the study of algebra. Students also begin to deal with informal proofs for theorems that will be proven more formally in high school.

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Grade 8 Adopted Washington State Mathematics Standards . . . . . April 28, 2008

8.1. Core Content: Linear functions and equations (Algebra)

Students solve a variety of linear equations and inequalities. They build on their familiarity with proportional relationships and simple linear equations to work with a broader set of linear relationships, and they learn what functions are. They model applied problems with mathematical functions represented by graphs and other algebraic techniques. This Core Content area includes topics typically addressed in a high school algebra or a first-year integrated math course, but here this content is expected of all middle school students in preparation for a rich high school mathematics program that goes well beyond these basic algebraic ideas.

8.2. Core Content: Properties of geometric figures (Numbers, Geometry/Measurement)

Students work with lines and angles, especially as they solve problems involving triangles. They use known relationships involving sides and angles of triangles to find unknown measures, connecting geometry and measurement in practical ways that will be useful well after high school. Since squares of numbers arise when using the Pythagorean Theorem, students work with squares and square roots, especially in problems with two- and three-dimensional figures. Using basic geometric theorems such as the Pythagorean Theorem, students get a preview of how geometric theorems are developed and applied in more formal settings, which they will further study in high school.

8.3. Core Content: Summary and analysis of data sets (Algebra, Data/Statistics/Probability)

Students build on their extensive experience organizing and interpreting data and begin to apply statistical principles to analyze statistical studies or short statistical statements, such as those they might encounter in the newspaper, on television, or on the Internet. They use mean, median, and mode to summarize and describe information, even when these measures may not be whole numbers. Students use their knowledge of linear functions to analyze trends in displays of data. They create displays for two sets of data in order to compare the two sets and draw conclusions. They expand their work with probability to deal with more complex situations than they have previously seen. These concepts of statistics and probability are important not only in students' lives, but also throughout the high school mathematics program.

8.4. Additional Key Content (Numbers, Operations)

Students deal with a few key topics about numbers as they prepare to shift to higher level mathematics in high school. First, they use scientific notation to represent very large and very small numbers, especially as these numbers are used in technological fields and in everyday tools like calculators or personal computers. Scientific notation has become especially important as "extreme units" continue to be identified to represent increasingly tiny or immense measures arising in technological fields. A second important numerical skill involves using exponents in expressions containing both numbers and variables. Developing this skill extends students' work with order of operations to include more complicated expressions they might encounter in high school mathematics. Finally, to help students understand the full breadth of the real-number system, students are introduced to simple irrational numbers, thus preparing them to study higher level mathematics in which properties and procedures are generalized for the entire set of real numbers.

Grade Eight Performance Expectations . . . . . .

Linear Functions and Equations (algebra)

8.1.A **Solve one-variable linear equations.** (7.1.E)

8.1.B **Solve one- and two-step linear inequalities and graph the solutions on the number line.**

8.1.C **Represent a linear function with a verbal description, table, graph, or symbolic expression, and make connections among these representations.**

8.1.D **Determine the slope and y-intercept of a linear function described by a symbolic expression, table, or graph.**

8.1.E **Interpret the slope and y-intercept of the graph of a linear function representing a contextual situation.**

8.1.F **Solve single- and multi-step word problems involving linear functions and verify the solutions.**

8.1.G *Determine and justify whether a given verbal description, table, graph, or symbolic expression represents a linear relationship.*

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Properties of Geometric Figures (numbers, geometry/measurement)

8.2.A **Identify pairs of angles as complementary, supplementary, adjacent, or vertical, and use these relationships to determine missing angle measures.**

8.2.B **Determine missing angle measures using the relationships among the angles formed by parallel lines and transversals.**

8.2.C *Demonstrate that the sum of the angle measures in a triangle is 180 degrees, and* **apply this fact to determine the sum of the angle measures of polygons and to determine unknown angle measures.**

8.2.D **Represent and explain the effect of one or more translations, rotations, reflections, or dilations (centered at the origin) of a geometric figure on the coordinate plane.**

8.2.E *Quickly* **recall the square roots of the perfect squares from 1 through 225 and estimate the square roots of other positive numbers.** **

8.2.F *Demonstrate the Pythagorean Theorem and its converse and* **apply them to solve problems.**

8.2.G **Apply the Pythagorean Theorem to determine the distance between two points on the coordinate plane.**

Summary and Analysis of Data Sets (algebra, data/statistics/probability)

8.3.A **Summarize and compare data sets in terms of variability and measures of center.** (7.4.C)

8.3.B **Select, construct, and analyze data displays, including box-and-whisker plots, to compare two sets of data.** (7.4.D)

8.3.C **Create a scatterplot for a two-variable data set, and, when appropriate, sketch and use a trend line to make predictions.**

8.3.D *Describe different methods of selecting statistical samples and analyze the strengths and weaknesses of each method.* (7.4.E)

8.3.E *Determine whether conclusions of statistical studies reported in the media are reasonable.* (7.4.E)

8.3.F **Determine probabilities for mutually exclusive, dependent, and independent events from small sample spaces.** (7.4.B)

8.3.G **Solve single- and multi-step problems using counting techniques and Venn diagrams and verify the solutions.**

Additional Key Content (numbers, operations)

8.4.A **Represent numbers in scientific notation, and translate numbers written in scientific notation into standard form.**

8.4.B **Solve problems involving operations with numbers in scientific notation and verify solutions.**

8.4.C **Evaluate numerical expressions involving non-negative integer exponents using the laws of exponents and the order of operations.** (6.2.D)

8.4.D *Identify rational and irrational numbers.*

Reasoning, Problem Solving, and Communication

8.5.A *Analyze a problem situation to determine the question(s) to be answered.* (7.6.A)

8.5.B **Identify relevant, missing, and extraneous information related to the solution to a problem.** (7.6.B)

8.5.C *Analyze and compare mathematical strategies for solving problems, and* **select and use one or more strategies to solve a problem.** (7.6.C)

8.5.D **Represent a problem situation,** *describe the process used to solve the problem, and* **verify the reasonableness of the solution.** (7.6.D)

8.5.E *Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.* (7.6.E)

8.5.F *Apply a previously used problem-solving strategy in a new context.* (7.6.F)

8.5.G **Extract and organize mathematical information from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.** (7.6.G)

8.5.H *Make and test conjectures based on data (or information) collected from explorations and experiments.* (7.6.H)