WRITTEN RESPONSE TO MATH

ENHANCING MATH INSTRUCTION

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INTRODUCTION

There are very few people who are indifferent about their skillfulness in math. People are either confident in their math skills or insecure about them. Why is that? When it comes to literacy, we seem to produce people who feel good (or at least somewhat capable) about themselves as readers and/or writers. What is the difference in literacy learning that, for many people, makes them feel more capable than when they're learning math? What is the difference in thinking? In reading and writing instruction we are taught different strategies and processes that we can use to think about a story even before we open a book or take pen to paper. (New Zealand Ministry of Education, 1992) (New Zealand Ministry of Education, 1997) For example, before we read we may check to see if we know the author and what he or she has written before. We think about the title and make predictions as to what we might find as we dig into the story. In writing we are taught to plan or brainstorm. We feel free and confident to revise, proofread, and edit. After years of instruction we internalize these reading and writing strategies and processes so we are able to utilize them effortlessly. Can the same type of thinking be used to develop mathematical literacy? What kind of thinking is required before or as we delve into math problems?

Different Kinds of Thinking

At any given time, people are engaged in narrative, expository and/or mathematical thought. Because these broad patterns make up the way we think, they also have an impact on the way we communicate. (Mooney, 2001) Whether we're thinking, speaking, reading, writing, or working with numbers, we weave in and out of these broad patterns of thought and communication. Teachers need to understand these broad patterns and how they can be used to advance student learning. By incorporating narrative, expository, and mathematical processes as part of their instruction, teachers can frame any content or curriculum by setting the purpose for students. This focus on the why behind the skills can help teachers create opportunities for students to apply their learning. It's this application of learning that supports students in making stronger connections between content areas and grade levels.

Math as a Thinking Process

Like the thinking skills inherent in both the reading and writing process, mathematics is a thinking process that leads to more effective and efficient problem solving—just knowing traditional algorithms isn't enough to use and apply math knowledge in a variety of contexts. The National Council of Teachers of Mathematics (NCTM) has outlined a math process through a series of standards (Process Standards, 2011). These standards are the rationale behind many math initiatives such as Common Core State Standards (Common Core State Standards Initiative, 2010).

Problem Solving

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts

- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs
- Select and use various types of reasoning and methods of proof

Communication

- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others;
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

Representation

- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena

So, what does a thinking or problem-solving process look like in math instruction? How do we teach students to think mathematically? The following table outlines a simplified structure to help teachers think about what a math process would look like in their classroom. It contains essential math skills and the related thinking needed for students to work through math problems. Note how many of the *essential math skills* and *mathematical thinking* connect to the NCTM math process standards.

By thinking about algorithms and concepts in terms of essential math skills, teachers can anticipate potential issues, better explain how to approach the problem and/or give more appropriate feedback. For example,

Essential Math Skills	Mathematical Thinking
Reasoning	What is the problem asking me to do? What actions do I see unfolding? Can I talk about the problem in my own words?
Observing/Identifying	What do I need to be attending to? What is the important information I need?
Strategizing	How will I find the answer? What do I know that will help me solve the problem? • Have I seen a problem like this before? • What does this problem remind me of? What calculations or operations do I need to consider?
Estimating	What would be a reasonable answer? How will I know if I'm close or on the right track?
Determining the Answer (Solution)	What is my answer? Is my answer close to what I thought it would be?
Articulating/Explaining	Can I justify my actions or decisions? Can I talk or write about the HOW and WHY of I what I did to solve the problem?

when looking over a math problem to give students, teachers need to be asking:

- How would I expect my students to visualize the problem? What is the action being described in the problem?
- What are the important words or information my students need to attend to in order to visualize, estimate or solve the problem?
- What are the possible strategies my students could use to solve the problem?
- How do I want my students to articulate the HOW and WHY of this problem?

Understanding how essential math skills can impact the student's thinking and problem-solving ability prior to the instruction of skills or concepts will help teachers deliver more effective instruction. In order to achieve this it is essential that teachers have an instructional tool to help them develop the mathematical thinker and problem solver. The Math Response Journal is such a tool.

THE MATH RESPONSE JOURNAL

Incorporating Written Response to Math in the Classroom

With processing skills in mind, many states and districts are requiring students to write across content areas. They understand the value of writing in determining what a student knows in any given subject matter. The focus of instruction in today's classroom is no longer just about having the "right" answer but how the answer was determined—the process the student went through. With this in mind, it is essential that written response become an important instructional practice within our classrooms, and math is no exception. Having students respond to math questions or problems, both orally and in writing, requires that students articulate the thinking they used as they worked through any math problem. Understanding the HOW and WHY of mathematical problem-solving, not only helps students talk or write about their problem-solving, but enhances the student's ability to understand math problems or concepts.

Written response is also a very effective way for teachers to assess where a student may be having trouble with a particular math problem, concept, procedure, and/or reasoning issue. Because teachers can see where there is partial understanding or where understanding breaks down, written response allows teachers to move their instruction beyond "getting the right answer" to a focus that is more meaning-based. Understanding becomes the focus of instruction, not the procedure or algorithm. Written response adds another dimension to the math instruction that goes on within the classroom. Let's look at how the Math Response Journal supports this kind of teaching and learning. (See pages 8 and 9 for a breakdown of how the journal supports essential math skills)

Using the Math Response Journal: A Classroom Example

Because the Math Response Journal was designed to help teachers develop a student's mathematical thinking, each section of the journal supports a student's ability to think through problems. The first two sections in the math journal, ***Write the question or problem*** and ***Show your work*** help support a student's ability to make sense of the problem. The grid provides a space for students to record their thinking. It creates an opportunity for students to think through the question and make decisions about what strategies might be helpful in solving the problem. The grid also offers space which allows the student to organize the information and calculate the answer.

The ***Explain HOW and WHY*** section is an area for students to practice articulating their thinking and problem-solving. Not only do students talk about the steps they took in this section, they also need to record the reasoning behind each step. It's this articulation that reinforces mathematical understandings and allows students to write/record/sound like mathematicians. A checklist at the end of every math journal entry reinforces the purpose of each section. (See Math Response Journal) In the following teaching scenario a teacher demonstrates how she uses the Math Response Journal in her classroom.

A Fourth Grade Lesson

Problem-solving in the Math Response Journal is a regular component of the math instruction in this

fourth grade classroom. The teacher uses the journal to assess her students' understandings of math concepts and skills. The Math Response Journal allows the teacher to see what her students understand at any given time. Whether it's prior to the introduction of a new skill or for on-going checks during the teaching of a unit, the Math Response Journal is a good way for teachers to check on understanding. In this lesson, the teacher wanted to see if her students could visualize the actions being asked in a multistep problem. With the whole class listening, the teacher refers the students to the problem on the board.

> Boris Bookworm just bought bookcases for his room. Each bookcase has four shelves and can hold eight books on a shelf. He has four bookcases in the room and they are filled. How many books does Boris have in his room?

Teacher: What is the question the problem is asking you to answer?

Student: How many books does Boris have in his room?

Teacher: So, what is the problem asking you to do?

Student: It wants you to find the number of books in Boris' room.

Teacher: Okay, if that's what the problem is asking, what's the important information you need to know from the problem?

Student: 4 bookcases, 4 shelves, and each shelf has 8 books on it.

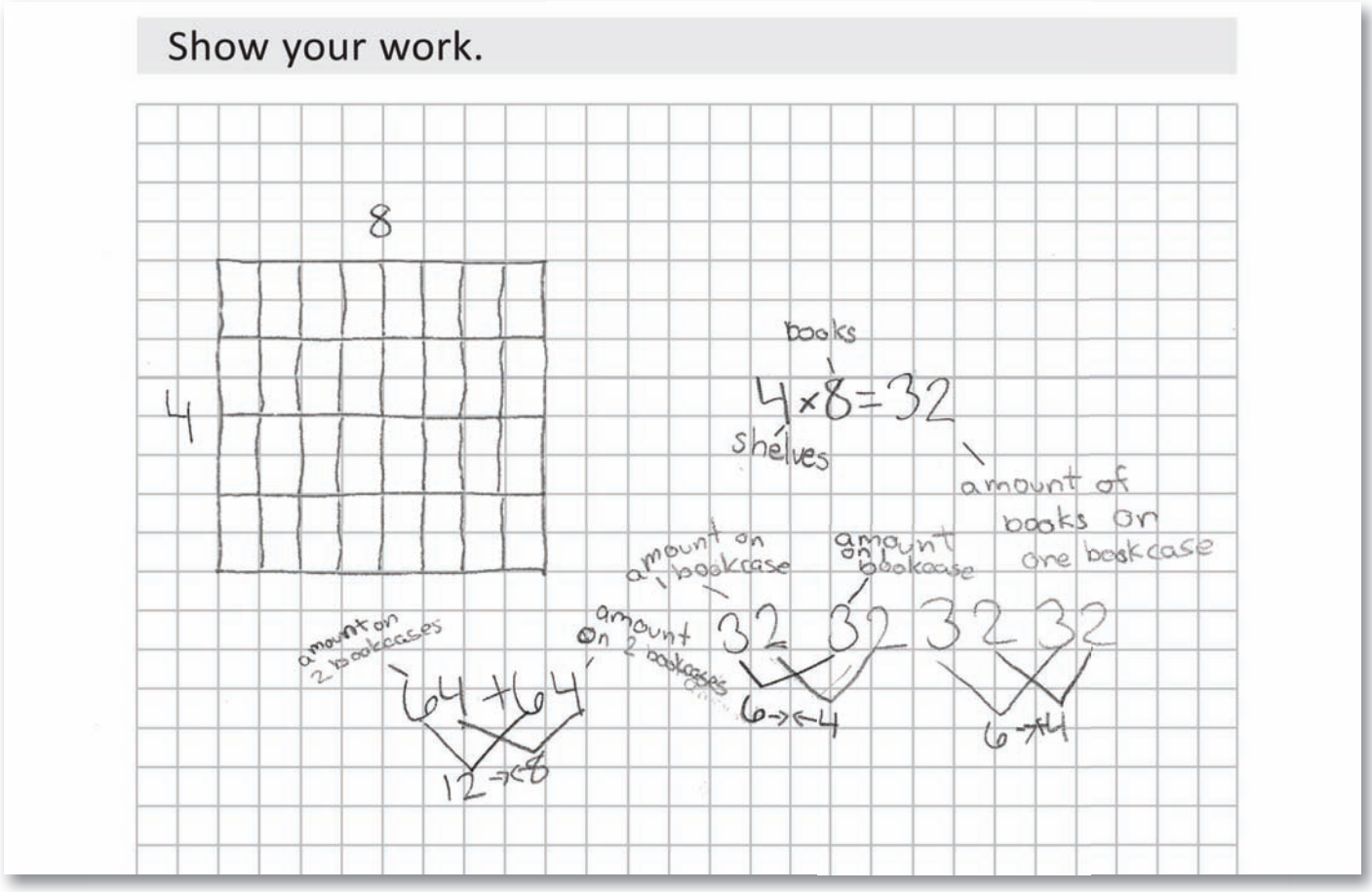
Teacher: Yes. With this information can you get a picture of what his bookcases look like?

Student: Yes!

Teacher: Remember, before you begin to figure out the problem you need think about what a reasonable answer would be so you'll know if your answer is right.

Student: I think 20 books because all these numbers together would be about 20.

Teacher: Is that for one bookcase?



Student: I think it will be more than 20 because one bookcase has 4 shelves with 8 books on it. That's more than 20. I think with all the bookcases there will be more than 100.

Student: I think it will be about 120 books. If there were 15 shelves with 8 books on each shelf there would be 120 books.

Teacher: Can you explain how you got 120 books?

Student: First, I knew there were 4 shelves on 4 bookcases so that's 16 shelves. I rounded down to 15 and multiplied it by 4 because I know 15 times 4 is 60.

Teacher: Where did the 4 come from?

Student: It's half of 8. There are 8 books on each shelf and knew that 15 times 4 is 60. Then I added two 60s and got 120. So there has to be more than 120 books.

Teacher: We have a big range of estimates for this problem. You've heard what your friends are thinking. Which estimate sounds right?

Student: I don't think 20 is right. I think 120 is a good answer.

Teacher: Alright, I think 120 is a reasonable answer too. Go ahead and write the problem down. Solve this problem in a way that makes sense to you. Label the parts of your solution to help you keep track of and share your thinking when you get to the ***How and Why*** section.

At a later time, the teacher takes a look at several students' responses to get a sense of where her students are in relation to her focus. At that point the teacher asks students to take out their journals and share their responses with the class. The following journal entry is an example taken from one of the students in the classroom.

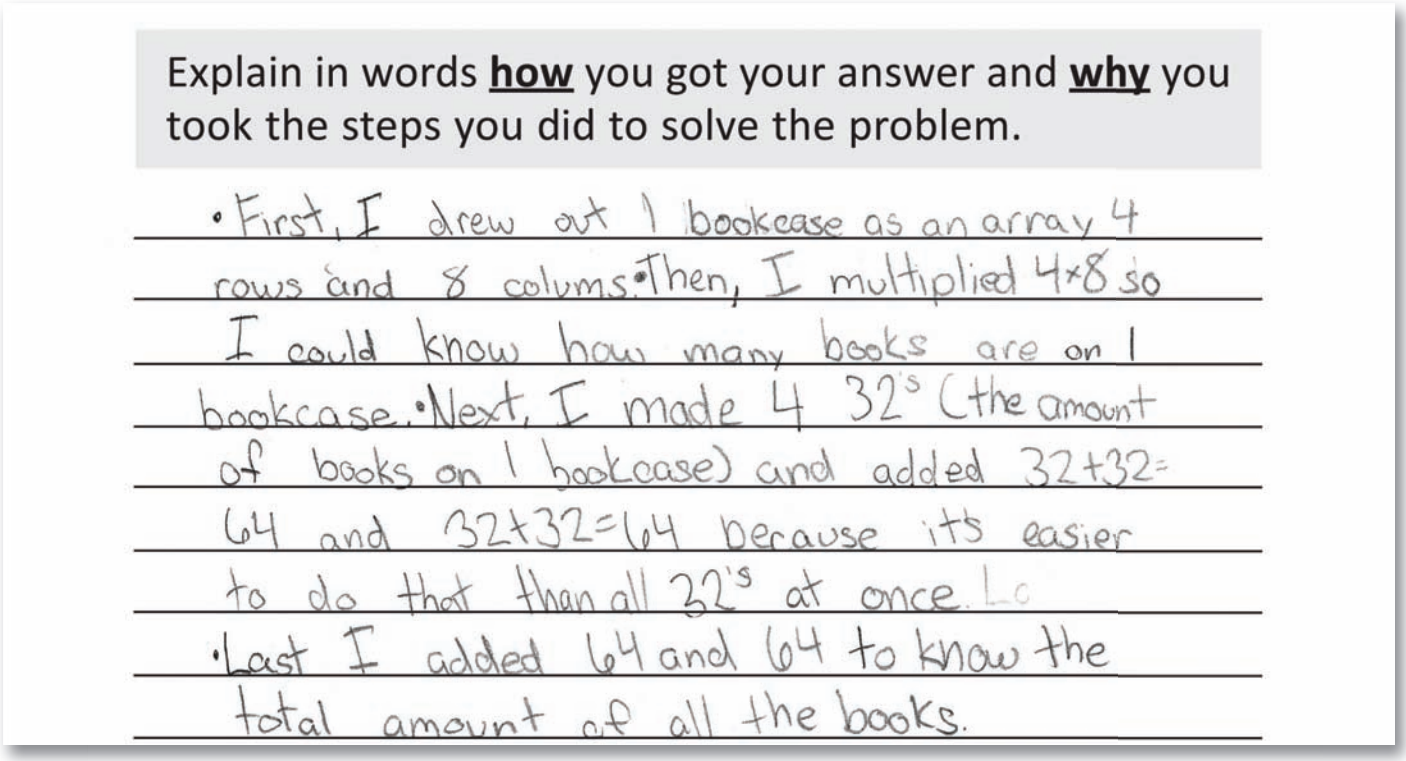
Teacher: Marina, would you come up to the document camera and share your response? How did you get started solving this problem?

Marina: First, I made a 4x8 array to show the books on one bookcase and found there are 32 books on one bookcase. (see page 4)

Teacher: Then how did you finish the problem?

Marina: Well, there are 4 bookcases so I needed to find 4 32s. 32+32=64 and 64+64=128.

Teacher: Who solved this in another way? Sandra, how did you get started?



Explain in words **how** you got your answer and **why** you took the steps you did to solve the problem.

• First, I drew out 1 bookcase as an array 4 rows and 8 colums. Then, I multiplied 4×8 so I could know how many books are on 1 bookcase. • Next, I made 4 32's (the amount of books on 1 bookcase) and added 32+32= 64 and 32+32=64 because it's easier to do that than all 32's at once. • Last I added 64 and 64 to know the total amount of all the books.

Sandra: I knew there were 16 shelves total in the room because each bookcase had 4 shelves and there were 4 bookcases. So 4x4=16.

Teacher: Once you knew there were 16 shelves, how did you complete the problem?

Sandra: I multiplied the sixteen shelves by eight books on each shelf. I broke up the 16 to make it easier. First I found 10x8=80 and then 6x8=48. Then, I added these parts back, 80+48=128. So there are 128 books in the room.

Teacher: So you both got 128. Does that answer seem reasonable to you?

Student: We thought it was going to be a little more than 120 and it was.

This teacher uses the Math Response Journal as a vehicle for higher mathematical thinking. She understands that students need the opportunity to articulate their understandings as they learn. When she uses questions such as, "What makes this a reasonable answer to you?" the teacher demonstrates she values their thinking and knows that they are capable. She also knows that students learn best from each other. By hearing what others have done and their thinking about it, students can extend their own thinking and be more flexible in how they approach their own mathematical problem-solving.

PLANNING FOR INSTRUCTION

Written Response Questions

A critical aspect of written response is the question or the problem. Written response questions can come from many resources, such as a teacher manual, but choosing a question that yields the most useful information about student learning is essential. There are two types of written response questions—short and long. In short written responses the focus is on the skill and showing the work. In long (or extended) written response, the skill is still important but there is also a focus on the strategy being used as well as the student's ability to articulate what s/he did. (see page 5) Both short and extended written responses have their place in a math setting but, depending on what the teacher is measuring or assessing, knowing when to use them can make all the difference to classroom instruction. The following state release test items (ISBE, 2011) are examples of both short and extended written responses. No matter where math written response questions come from, designing them to resemble questions from state tests helps make a strong link between instruction and assessment. The results will be evident in both understanding and increased student achievement.

The first two numbers in a pattern are 1 and 3.

1, 3, ___, ___, ___

- Explain a rule you could use to continue the pattern.
- Use your rule to find the next three numbers in the pattern.

Short Written Response

Five friends bought a total of two pizzas. Each pizza was cut into 12 equal pieces. The amount of one whole pizza each person ate is shown below.

Joe ½ Mary ¼ Kim ⅓ Bill ⅓ Sue ¼

How many slices of pizza were not eaten by these five friends?

Show all your work. Explain in words how you found your answer. Tell why you took the steps you did to solve the problem.

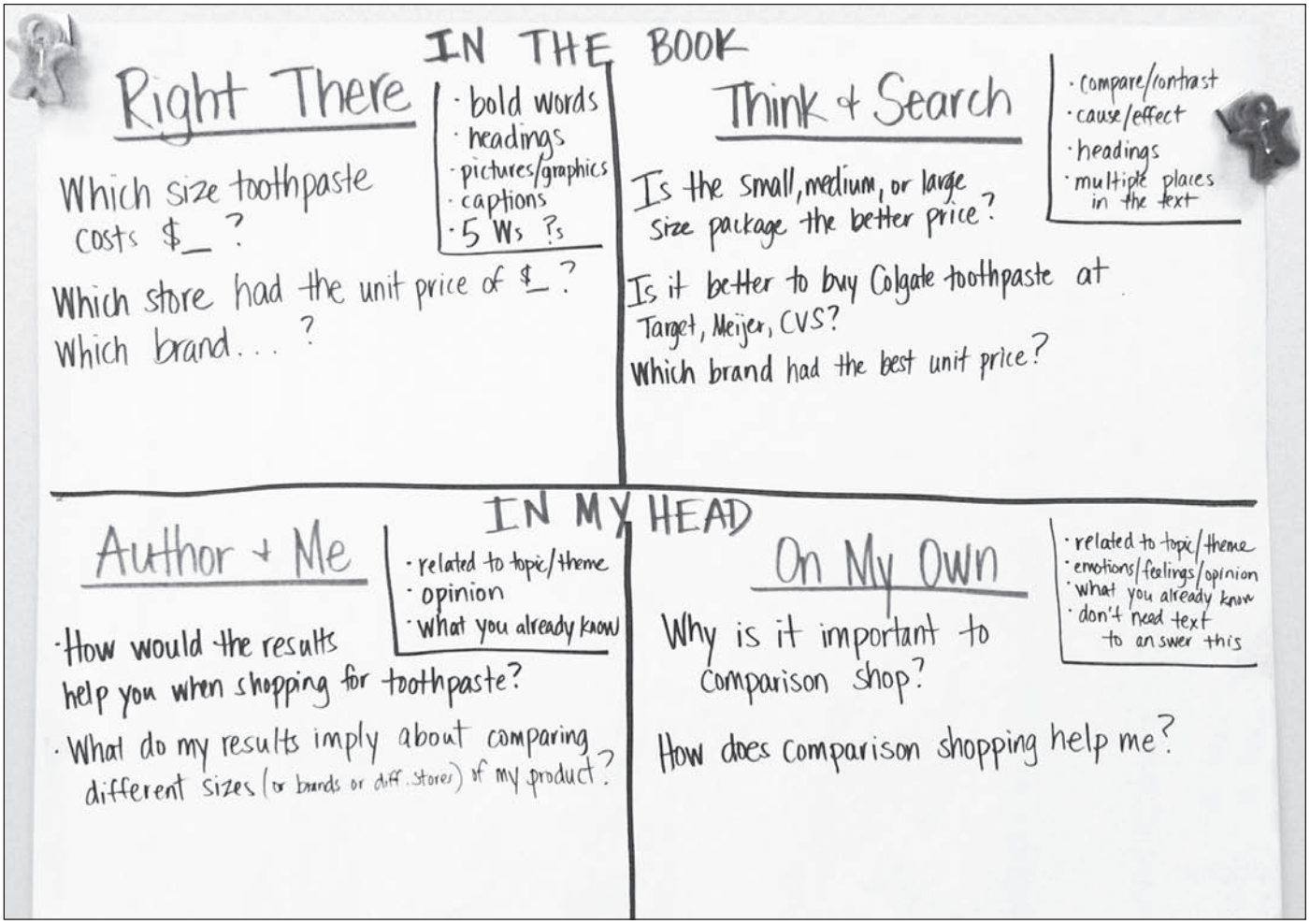
Long or Extended Written Response

Helping students understand how to interpret questions in order to find the answer is also a critical component in the development of the mathematical thinker. Just as in the two different questions above, the two answers will require a different kind of problem-solving. Knowing

where to find the information critical to answering the question will help students better formulate their answers. In the picture below, a seventh grade teacher is helping her students think through math questions/problems. She and her students are reviewing their written responses in their Math Response Journals and analyzing how the responses are different. Her school uses QAR: Question Answer Relationship which looks at different types of reading questions, from literal to analytical, and how that affects the answers. (Raphael, 2006) The teacher is applying that same reading work to the written response questions she uses in math. In doing so, not only is the teacher helping students make connections from their literacy instruction to math but, she is supporting higher level thinking skills (problem-solving) and their ability to explain their work. Note the smaller boxes in each of the four sections that give students ideas on either where to find information related to the kind of question (i.e. bold words, pictures/graphics) or ways to structure their response (i.e. compare/contrast, cause/effect).

CONCLUSION

Thinking about mathematics as a process, and not a series or set of algorithms to be memorized, can make a significant impact in both teachers' math instruction and students' learning. Teaching the how and why behind the math skills and concepts we use, not only deepens understanding, but can greatly affect student achievement. In order for this to occur, it is essential that teachers have the right instructional tools. The Math Response Journal is an instructional tool that supports both teachers and students in digging deeper into mathematical problem-solving. By using the Math Response Journal as an instructional resource, teachers will have an assessment sample of the strategies used by students and their ability to justify their reasoning for those strategies. The students, in turn, will become more flexible in how they approach the problems they encounter, as well as becoming more confident in their mathematical ability.

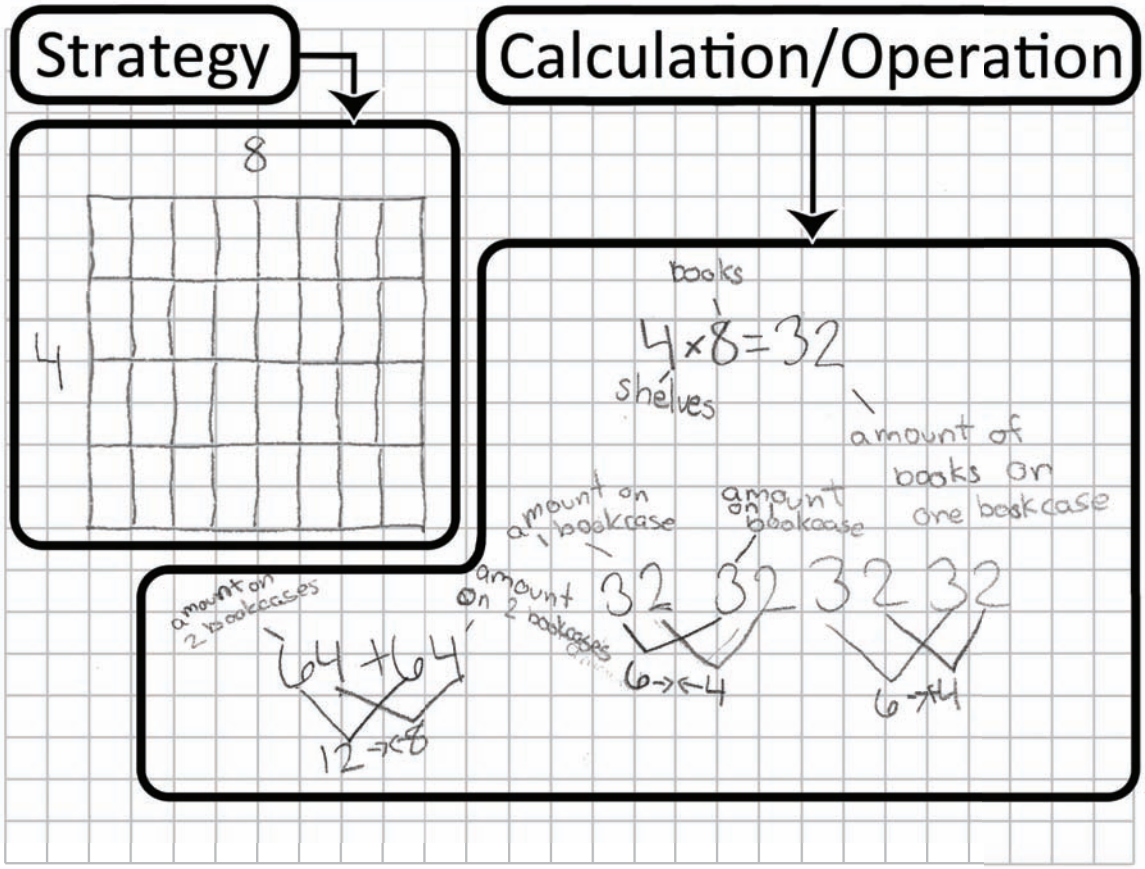


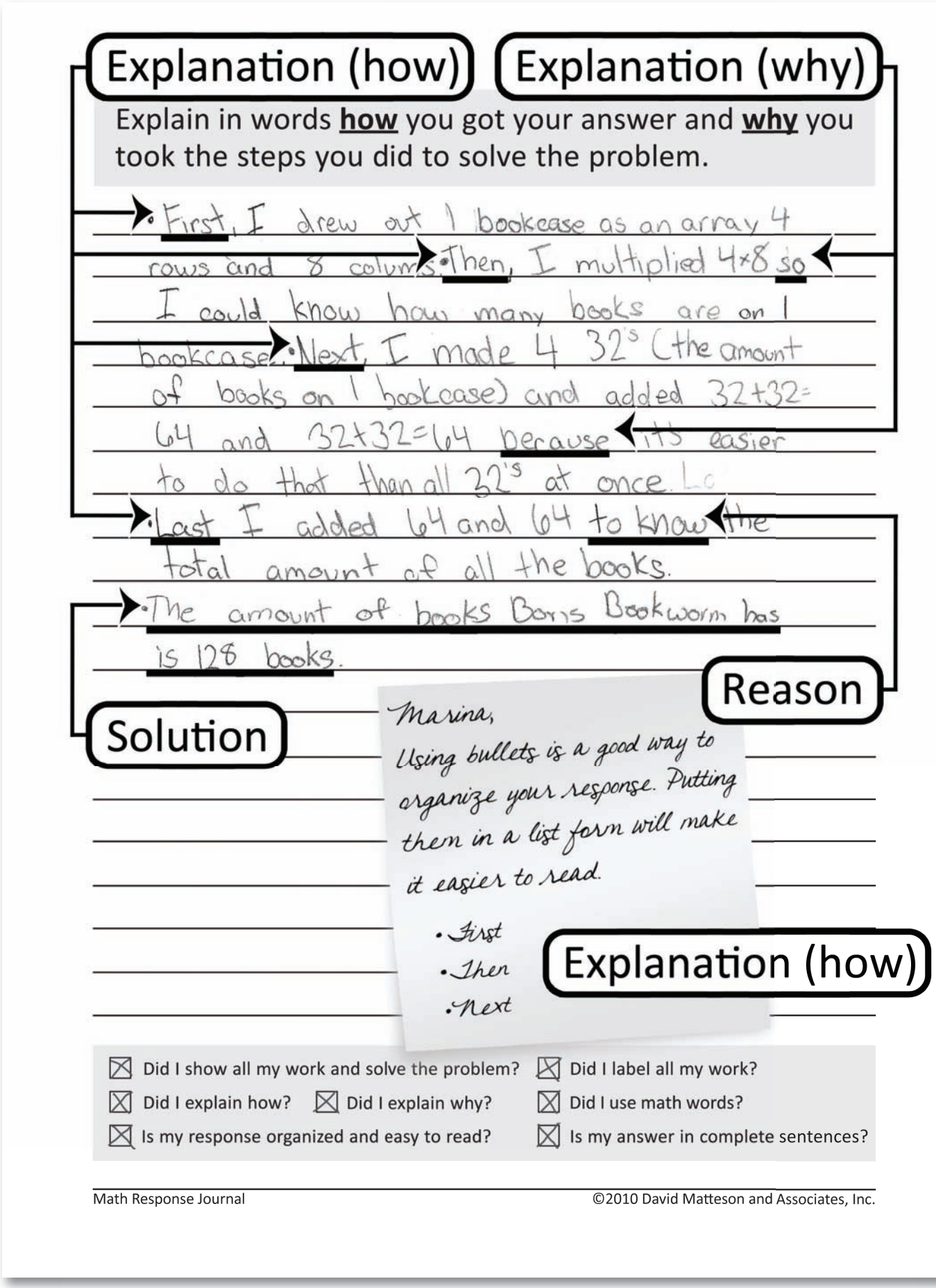
Example

Write the question or problem.

Boris Bookworm just bought bookcases for his room. Each book case has 4 shelves and can hold 8 books on a shelf. He has 4 bookcases in the room and they are filled. How many books does Boris have in his room?

Show your work.





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